UNSTEADY HEAT TRANSFER AT AN INSULATED SECTION OF A SURFACE

E. V. Kudryavtsev and K. N. Chakalev

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It is established that in unsteady heat transfer from a stream of gas to a plate with insulated sections (heat flux sensors), the heat-transfer coefficient depends on the thermal head or on the rate of change of the heat flux.

When it is necessary to study unsteady heat transfer of large surfaces without the possibility of changing the temperature distribution arbitrarily, the method of local insulated sections may be useful. At insulated sections at specified places on the surface, heat transfer is studied for an arbitrarily varying temperatures of the sections and a fixed temperature of the rest of the surface. The temperature of the sections can be measured by one of the methods of unsteady heat transfer [1].

The importance of the dimensions and thermophysical parameters of the bodies themselves in unsteady heat transfer was first pointed out in [1].

The characteristic features of unsteady heat transfer mentioned was investigated from various points of view in [2] and subsequent theoretical and experimental papers, and summarized in [3]. In the heating or cooling of bodies instantaneously moved into a fluid, or for a rapid change of boundary conditions, the heat-transfer coefficient is appreciably different from its value in a quasi-steady state.

The present article presents results of a theoretical and experimental study of heat transfer of an insulated section (as $Bi \to min.$) on a flat plate which is rapidly introduced into a stream of hot gas.

As the stream of gas moves from the main surface to an insulated section, the stepwise change in temperature of the surface leads to a change in the thermal boundary layer above it. Heat transfer at the surface of the section will depend on the heat-transfer conditions at the insulated section of the plate surface.

For given heat-transfer conditions it is easy to obtain the solution of the boundary layer equations in the quasi-steady approximation for constant thermophysical characteristics of the stream and its temperature as it leaves the surface. The differential equations for the boundary layer are linear, and by using Duhamel's theorem, the solution is obtained as the sum of the solutions for an isothermal plate and a plate with an insulated section at which there is no heat transfer [4].

We consider laminar and turbulent boundary layers and the case of a turbulent layer when the surface of the section of the plate is small and the change of temperature in the boundary layer occurs completely or mainly in the laminar sublayer.

In the mathematical formulation of the problem we assume the following conditions:

1) the dynamic turbulent boundary layer is developed directly from the leading edge of the plate;

2) steady surface flow of fluid with constant properties; the fluid velocity is low; the dissipation of energy is neglected;

3) the quasi-steady approximation is used for the thermal boundary layer;

4) the Reynolds analogy is satisfied for the sublayer and turbulent region;

5) a two-layer model of the dynamic turbulent boundary layer is used;

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6) the solutions for laminar and turbulent layers are constructed in the same way be using momentum and energy integrals; the only difference is in the velocity and temperature distribution laws: parabolic for laminar flow; one-seventh power law for the velocity and temperature distributions in a turbulent layer; linear for the sublayer with a change in temperature in the sublayer only; the thickness of the whole boundary layer is determined from the momentum integral, with a one-seventh power law for the variation of velocity;

7) the Blasius relation was used for a turbulent boundary layer;

$$\tau_w = 0.022\rho u_\infty^2 \left(\frac{\nu}{u_\infty \delta_T}\right)^{\frac{1}{4}};$$

8) the boundary conditions for the temperature of the surface vary in a stepwise manner; the temperature of the surface downstream from a step varies with time:

$$T(x,\ \Delta) = T_\infty;\quad T(x,\ 0) = T_w.$$

For a power law variation of the temperature distribution over the thickness of a layer

$$\frac{\partial T(x,\ \Delta)}{\partial y} = 0 \text{ and } \frac{\partial^2 T(x,\ 0)}{\partial y^2} = 0.$$

Influence functions of the prehistory of the boundary layer $[1 - (\xi/x)^m]^n$, the thickness Δ of the thermal boundary layer, and the heat flux q to the section of the surface were determined from the solution.

For the thermal boundary layer, which is completely contained with the laminar sublayer, influence functions were obtained for the initial unheated section, the thickness of the thermal boundary layer, and the heat flux for the section of the plate:

$$\left(1 - \frac{\xi}{x}\right)^{1/3}, \tag{1}$$

$$\Delta = 6.83 \mathrm{Re}_x^{-3/5} \mathrm{Pr}^{-1/3} x \left(1 - \frac{\xi}{x}\right)^{1/3}, \tag{2}$$

$$q = 0.141 \frac{\lambda}{x} \mathrm{Re}_x^{3/5} \mathrm{Pr}^{1/3} \left(1 - \frac{\xi}{x}\right)^{1/3} (T_\infty - T_w), \tag{3}$$

the heat flux for the isothermal insulated section

$$q = 0.0295 \frac{\lambda}{x} \mathrm{Re}_x^{4/5} \mathrm{Pr}^{3/5} \left[1 - 5\mathrm{Re}_x^{-1/5} \mathrm{Pr}^{-4/15} \frac{T_w - T_0}{T_\infty - T_0} \left(1 - \frac{\xi}{x}\right)^{-1/3}\right] (T_\infty - T_0). \tag{4}$$

On the basis of the expressions obtained, an analysis was made of the possibility of realizing such conditions when the thermal boundary layer above the section of the surface is nearly or completely within the laminar sublayer. As a result, it was established that such a case did not occur in an air jet under the conditions of the experiment with the gas-dynamic equipment, since the insulated section of the plate surface close to the leading edge was rather large, and the thermal boundary layer increased rapidly.

Here it is necessary to pay attention to the characteristics of the formulation of the problem. Equation (4) clearly will not hold at points very close to stepwise temperature changes. Thus, Eq. (4) shows that as the temperature step is approached the heat flux tends to infinity. This is accounted for by the assumption of the contact of two media with different temperatures. To obtain a more accurate solution it is necessary first to take account of the thermal conductivity of the insulated section of the surface, i.e., to consider the adjoint problem, taking account of the propagation of heat in the fluid and the section in two directions.

Thus, for small sections of the surface far from the beginning of the boundary layer, analysis alone may be insufficient. In this case the relations obtained and the usual justification for using the method of local sensors under specific conditions must be verified experimentally.

Solutions for laminar and developed turbulent boundary layers are easily obtained by this same scheme. The main difference will be in the assumed temperature and velocity distribution laws and accordingly in the solutions of the integral equations.

Detailed solutions are given in [4].

The solution obtained for the heat flux can be written in the general form

$$q = A\left[1 - \frac{T_w - T_0}{T_\infty - T_0} B\right](T_\infty - T_0), \tag{5}$$

where for a laminar boundary layer

$$A = A_{\rm L} = 0.332 \frac{\lambda}{x} {\rm Re}_x^{1/2} {\rm Pr}^{1/3}, \tag{6}$$

$$B = B_{\rm L} = \left[1 - \left(\frac{\xi}{x}\right)^{3/4}\right]^{-1/4} \tag{7}$$

and for a turbulent boundary layer

$$A = A_{\rm T} = 0.0295 \frac{\lambda}{x} {\rm Re}_x^{4/5} {\rm Pr}^{3/5}, \tag{8}$$

$$B = B_{\rm T} = \left[1 - \left(\frac{\xi}{x}\right)^{9/10}\right]^{-1/9}. \tag{9}$$

An expression can be obtained from Eq. (5) for the heat-transfer coefficient for a section of the surface with a time-varying temperature

$$\alpha = \frac{q}{T_\infty - T_w} = AB + A(1 - B)\frac{\vartheta_0}{\vartheta} \tag{10}$$

$$\alpha = A\frac{\vartheta_0}{\vartheta} - AB\frac{\vartheta_0 - \vartheta}{\vartheta}, \tag{11}$$

where $\vartheta_0 = T_\infty - T_0$, $\vartheta = T_\infty - T_w$.

Thus, the solution of the boundary layer equation in the quasi-steady approximation gives an inverse or hyperbolic variation of the heat-transfer coefficient with the temperature difference ϑ. If the temperature of a section of the surface T_w and that of the rest of the surface T_0 are equal, the heat-transfer coefficient will be equal to A, i.e., to its value under steady conditions for an isothermal plate.

On the other hand, for small Biot numbers the heat-conduction equation for an insulated section of the surface has the form

$$c\rho R\frac{d\vartheta}{d\tau} + AB\vartheta + A(1 - B)\vartheta_0 = 0, \tag{12}$$

and by using (10) and (11) its solution can be written as

$$\frac{\vartheta}{\vartheta_0} = 1 - \frac{1}{B}\left(1 - \exp\left[-\frac{AB}{c\rho R}\tau\right]\right). \tag{13}$$

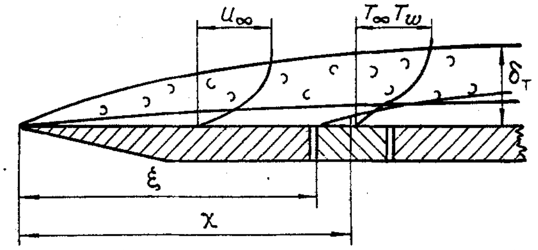

Fig. 1. Schematic diagram of experimental model.

The heat flux is

$$q(\tau) = \vartheta_0 A \exp\left[-\frac{AB}{c\rho R}\,\tau\right]. \tag{14}$$

The above shows that the rate of heating of a section of the plate is determined by heat transfer from the stream, and the external data are controlling in this case. The quantity B characterizes the nonuniformity of the temperature distribution over the surface; its value as $\tau \to \infty$ determines the limiting temperature of a section of the plate when

$$B\left(1-\frac{\vartheta}{\vartheta_0}\right) = 1, \quad \text{or} \quad B = \frac{\vartheta_0}{\vartheta_0 - \vartheta}. \tag{15}$$

Equation (13) can be written in the dimensionless form

$$\frac{\vartheta_0 - \vartheta}{\vartheta_0} = \frac{T_w - T_0}{T_\infty - T_0} = \frac{1}{B}\left(1 - \exp[-\mathrm{Bi}'\,\mathrm{Fo}]\right), \tag{16}$$

where $\mathrm{Bi}' = BAR/\lambda$, and $\mathrm{Fo} = a\tau/R^2$.

Thus, the heating process for a section will be characterized by Bi'Fo, which is analogous to the homochronous number in which, in addition to the dimensionless time and the heat-transfer coefficient for an isothermal plate A, there enters the dimensionless quantity B which characterizes the variation of temperature with coordinate (or over the section) and takes account of the previous history of the development of the thermal boundary layer.

The calculated results were compared with experimental data obtained with apparatus consisting mainly of a powerful three-phase alternating current electric arc heater in an air stream [5]. The diameter of the cross section of the jet at the channel jet where the temperature and velocity are constant was large enough for performing experiments with a model plate placed 5-10 mm from the exit of the stabilizing channel. The temperature in the jet core was $T_\infty = 2700$°K, the velocity was $u_\infty = 170$ m/sec, and the pressure was 1 abs. atm. The plate was 60 mm wide, the distance to the insulated section was $\xi = 52$ mm, and $x = 58$ mm (Fig. 1). The surface temperature of the insulated section was assumed equal to the initial temperature of the whole model $T_0 = 280$°K. By making a small ridge 0.2 mm high at the sharp edge of the plate, the boundary layer could be considered turbulent for the important characteristics of the flow from the very beginning of the plate. The heated insulated section was a cylinder of electrolytic copper 15 mm in diameter and 16.9 mm high. The surface of the section was insulated with a porcelain ring and asbestos cord, and the temperature was measured with Chromel—Copel thermocouples and recorded on an electronic potentiometer.

The points in Fig. 2 represent measured values at the temperature of the insulated section of the plate, and the curve is a plot of Eq. (16). The value of A used in the calculation was determined from an experiment in the region where T_W is equal or close to T_0, and where on the basis of Eq. (10) its value is equal to the heat-transfer coefficient for flow along an isothermal plate.

Figure 3 shows the relative heat-transfer coefficient α/α_0 as a function of the dimensionless temperature ϑ/ϑ_0, calculated by Eq. (10).

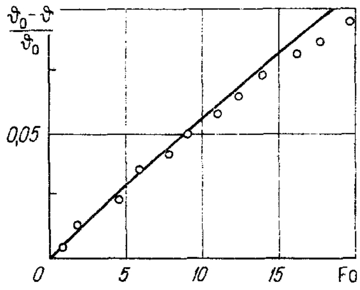

Fig. 2

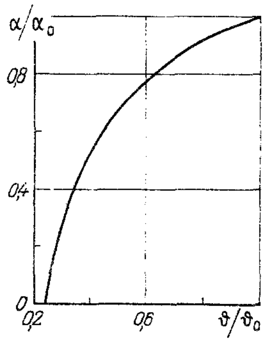

Fig. 3

Fig. 2. Theoretical and experimental time dependence of the temperature of a thermally insulated section of a plate in a stream of hot air; the points are experimental values, and the curve was calculated from Eq. (16).

Fig. 3. Relative heat-transfer coefficient α/α_0 as a function of dimensionless temperature ϑ/ϑ_0.

The calculated and experimental variations of the temperature of the insulated section as a function of the time (Fo number) are in satisfactory agreement. Their difference at higher temperatures is apparently related to the increase in heat loss through the insulation, radiation, and a change in the thermophysical properties of the gas and the material of the section under study.

The calculations show that even for a small rise in temperature of the section there is an appreciable change in its heat transfer with time; under our experimental conditions, as $\tau \to \infty$ the change reached 23%, a value determined by the magnitude of B.

Thus, in using insulated sections as heat flux sensors in stuying unsteady heat transfer of a surface, it is necessary to take account of the dependence of the heat-transfer coefficient on the temperature drop in both the calculations and formulation of experiments.

NOTATION

Bi, Biot number; τ_w, frictional shear stress at the wall; u_∞, velocity of incoming flow; ν, kinematic viscosity; λ, thermal conductivity; ρ, density; δ_T, thickness of turbulent boundary layer; x, running coordinate along surface of plate; ξ, coordinate of beginning of thermally insulated section; Re_x, local Reynolds number; Pr, Prandtl number; c, specific heat; a, thermal diffusivity; τ, time; Fo, Fourier number; R, thickness of insulated section; T_0, initial temperature of thermally insulated section and temperature of plate surface; T_∞, temperature of incoming flow; T_w, temperature of thermally insulated section; α_0, heat-transfer coefficient corresponding to the beginning of the heating of the insulated section; α, heat-transfer coefficient during the heating process; ϑ/ϑ_0, dimensionless temperature; A, local heat-transfer coefficient at isothermal surface; B, a quantity characterizing the temperature distribution over the surface of the plate.

LITERATURE CITED

1. E. V. Kudryavtsev, K. N. Chakalev, and N. V. Shumakov, Unsteady Heat Transfer [in Russian], Akad. Nauk SSSR, Moscow (1961), p. 160.
2. E. V. Kudryavtsev, "The use of unsteady thermal regimes to study heat transfer in airplane engines," Proc. TsIAM, Oborongiz, Moscow (1948), No. 140, p. 27.
3. B. M. Smol'skii and A. A. Sergeeva, "Unsteady heat transfer (review article)," Inzh.-Fiz. Zh., 17, No. 2 (1969).
4. W. M. Kays, Convective Heat and Mass Transfer, McGraw-Hill, New York (1966).
5. V. P. Motulevich, K. N. Chakalev, and V. I. Gudkov, "Methods of investigating heat and mass transfer of electrodes in a jet of low-temperature plasma," in: Magnetohydrodynamic and Electrophysical Characteristics of Streams of a Conducting Gas [in Russian], G. M. Krzhizhanovskii Power Engineering Institute (1973).